The Casimir effect at finite temperature in the presence of compactified universal extra dimensions

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Abstract

In this paper the Casimir effect for parallel plates at finite temperature in the presence of compactified universal extra dimensions is analyzed. We show the thermal corrections to the effect in detail. We investigate the Casimir effect for different size of universal extra dimensions.

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1. Introduction

More than 80 years ago the idea that our world has more than three spatial dimensions was put forward by Kaluza and Klein [1, 2]. They introduced an additional compactified dimension in order to unify gravity and classical electrodynamics. Recently, as one of the fundamental aspects, string theory needs seven additional spatial dimensions to unify the quantum mechanics and gravity. It is interesting that the order of the compactification scale of the extra dimensions is different in the branches of string theory. Some models of string theory expect that the typical size of compactified universal extra dimensions is of order $10^{-35}m$, which means that observing the effect is beyond our experimental reach of today and near future [3, 4]. In some approaches large extra dimensions were invoked for solving the hierarchy problem [5-8]. It is supposed that gauge fields are localized on a four dimensional brane, our real universe, and only a graviton can propagate in the extra space transverse to the brane.

The Casimir effect is a fundamental aspect of quantum field theory in confined geometries and has been a subject of extensive research [9-14]. The precision of the measurements has been greatly improved experimentally [15]. Now the Casimir effect becomes an experimentally powerful method for the study of new physics beyond the standard model. There are many cosmological uses of Casimir effect contributing with the right order of magnitude to the observed value of the cosmological constant [16-18]. A lot of topics related to the Casimir effect have been explored in the context of string theory [19-21]. More progresses of the effect were made to stabilize the extra dimensions [22, 23].

It is crucial but difficult to investigate how many dimensions our universe has and its structure theoretically and experimentally. If our spacetime has ten or more dimensions, we should observe the new phenomena related to the existence of the additional dimensions. Probing the possible existence and size of universal extra dimensions (UXDs) by means of Casimir effect attracts more attentions of the physical community [24, 25]. The Casimir force between two parallel plates in the presence of UXDs is investigated. The recent results show that the lower bound in energy is 300 GeV, which corresponds to a maximum size of extra dimension of about $10^{-9}nm$ [24]. There is an upper limit of $R \leq 10nm$ with one UXD [25].

Quantum field theory at finite temperature shares many of the effects. Thermal influence on the general Casimir effect can not be neglected in many cases [26-28]. It is essential to discuss the Casimir effect for the system consisting of two parallel plates in the presence of UXDs under a nonzero temperature environment. In this letter we consider some important and interesting arguments on the Casimir effect for the parallel plates at finite temperature with UXDs in detail. We predict that the thermal influence can not be omitted and the phenomena related to UXD is more manifest.

At different temperature and with various order of compactified extra dimensions of spacetime the description of the Casimir effect for parallel plates change greatly. Here we discuss the thermal corrections to the Casimir effect for the system by means of the Kaluza-Klein theory and finite-

temperature field theory [2, 3, 29, 30]. By regularizing the total energy, we find that the sign of Casimir energy for parallel plates in the world with one extra dimension is also positive at sufficiently high temperatures. The plates will repulse each other for their large enough distance. The phenomena related to Casimir effect with UXD become manifest. We try to test the size of UXD suggested in string theory. In this paper we derive the Casimir energy for parallel plates at finite temperature with UXD. We discuss the Casimir energy and Casimir force at different temperatures and with various size of extra dimension. Finally the conclusions are emphasized.

2. The Casmir energy for parallel plates at finite temperature in the spacetime with one extra dimension

In the Kaluza-Klein (KK) approach the Lagrangian density of a simple model of scalar field in (4+1)-dimensional spacetime takes the form,

$$\mathcal{L} = \frac{1}{2}\partial_A \Phi \partial^A \Phi \tag{1}$$

here $\Phi(x^A) = \Phi(x^\mu, y)$ is the field, $A = 0, 1, 2, 3, 5$ and $\mu = 0, 1, 2, 3$ belonging to four-dimensional coordinates and y to extra coordinate. According to the compactification of extra dimension on a circle, the field Φ can be expanded in the harmonics as follow,

$$\Phi(x, y) = \sum_{n=-\infty}^{\infty} \phi_n(x) e^{iny/L} \tag{2}$$

By substituting (2) into (1), the Lagrangian density (1) becomes,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_0 \partial^\mu \phi_0 + \sum_{k=1}^{\infty} (\partial_\mu \phi_k \partial^\mu \phi_k^* + \frac{k^2}{L^2}\phi_k \phi_k^*) \tag{3}$$

where L is the radius of UXD.

In finite-temperature field theories the imaginary time formalism can be used to describe the scalar fields in thermal equilibrium [26-30]. We introduce a partition function for a system in the presence of UXD,

$$Z = N \int_{period} \prod_k D\phi_k \exp[\int_0^\beta d\tau \int d^3x \mathcal{L}(\phi_\parallel, \partial_\mathcal{E} \phi_\parallel)] \tag{4}$$

where $\mathcal{L}$ is the Lagrangian density denoted as (3), N a constant and "period" means $\phi_k(0, \mathbf{x}) = \phi_\mathbf{k}(\tau, \mathbf{x})$, $k = 0, 1, 2, \cdots$. $\beta = \frac{1}{T}$ is the inverse of the temperature. The scalar fields ϕ_k satisfy the Klein-Gordon equations,

$$(\partial_\mu \partial^\mu - \frac{k^2}{L^2})\phi_k(x) = 0 \tag{5}$$

where $k = 0, 1, 2, \cdots$. The field confining between the two parallel plates satisfy the Dirichlet boundary conditions $\phi_k(x)|_{\partial\Omega} = 0$, $\partial\Omega$ positions of the plates. Following [26-28], the generalized zeta function reads,

$$
\begin{aligned}
\zeta(s;-\partial_E) &= Tr(-\partial_E)^{-s} \\
&= \int d^2k \sum_{m=1}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{n=0}^{\infty}[k^2+\frac{m^2\pi^2}{R^2}+\frac{n^2}{L^2}+(\frac{2l\pi}{\beta})^2]^{-s}
\end{aligned}
\tag{6}
$$

where R is the distance of the plates and $\partial_E = \frac{\partial^2}{\partial\tau^2}+\nabla^2$ with $\tau = it$. Furthermore, the function (6) can also be expressed in terms of Epstein zeta function E and Riemann zeta function ζ,

$$
\begin{aligned}
\zeta(s;-\partial_E) &= \pi^{3-2s}\frac{\Gamma(s-1)\zeta(2s-2)}{\Gamma(s)}\frac{1}{R^{2-2s}}+\pi\frac{\Gamma(s-1)}{\Gamma(s)}E_2(s-1;\frac{\pi^2}{R^2},\frac{1}{L^2}) \\
&+2\pi\frac{\Gamma(s-1)}{\Gamma(s)}E_2(s-1;\frac{\pi^2}{R^2},\frac{4\pi^2}{\beta^2})+2\pi\frac{\Gamma(s-1)}{\Gamma(s)}E_3(s-1;\frac{\pi^2}{R^2},\frac{1}{L^2},\frac{4\pi^2}{\beta^2})
\end{aligned}
\tag{7}
$$

the energy density of the model with thermal corrections and UXDs is,

$$
\begin{aligned}
\varepsilon &= -\frac{\partial}{\partial\beta}(\frac{\partial\zeta(s;-\partial_E)}{\partial s}|_{s=0}) \\
&= -\frac{\pi^{\frac{7}{2}}}{2R^3}\Gamma(-\frac{3}{2})\zeta(-3)-\frac{\sqrt{\pi}}{2}\Gamma(-\frac{3}{2})E_2(-\frac{3}{2};\frac{\pi^2}{R^2},\frac{1}{L^2}) \\
&+2^{\frac{3}{2}}\pi^2(\beta R)^{-\frac{3}{2}}\sum_{n_1,n_2=1}^{\infty}(\frac{n_2}{n_1})^{\frac{3}{2}}K_{\frac{3}{2}}(\beta\frac{\pi}{R}n_1n_2) \\
&+(2\pi^2)^{\frac{3}{2}}\beta^{-\frac{1}{2}}R^{-\frac{5}{2}}\sum_{n_1,n_2=1}^{\infty}n_1^{-\frac{1}{2}}n_2^{\frac{5}{2}}[K_{\frac{1}{2}}(\beta\frac{\pi}{R}n_1n_2)+K_{\frac{5}{2}}(\beta\frac{\pi}{R}n_1n_2)] \\
&+4\pi\sum_{k=0}^{\infty}\frac{8^{-k}(k+1)}{k!}\beta^{-k-2}\prod_{j=1}^{k}[9-(2j-1)^2]\sum_{n_1,n_2,n_3=1}^{\infty}n_1^{-k-2}(\frac{\pi^2}{R^2}n_2^2+\frac{1}{L^2}n_3^2)^{-\frac{k-1}{2}} \\
&\qquad\times\exp[-\beta n_1(\frac{\pi^2}{R^2}n_2^2+\frac{1}{L^2}n_3^2)^{\frac{1}{2}}] \\
&+4\pi\sum_{k=0}^{\infty}\frac{8^{-k}}{k!}\beta^{-k-1}\prod_{j=1}^{k}[9-(2j-1)^2]\sum_{n_1,n_2,n_3=1}^{\infty}n_1^{-k-1}(\frac{\pi^2}{R^2}n_2^2+\frac{1}{L^2}n_3^2)^{-\frac{k-2}{2}} \\
&\qquad\times\exp[-\beta n_1(\frac{\pi^2}{R^2}n_2^2+\frac{1}{L^2}n_3^2)^{\frac{1}{2}}]
\end{aligned}
\tag{8}
$$

where $K_\nu(z)$ is the modified Bessel function of the second kind. Now we introduce two dimensionless variables, the scaled temperature and ratio of plates distance to UXD size respectively,

$$
\begin{aligned}
\xi &= TL \\
\mu &= \frac{R}{L}
\end{aligned}
\tag{9}
$$

By regularizing the expression, we obtain the Casimir energy for parallel plates at nonzero temperature with UXD as follow,

$$\varepsilon_C = -\frac{1}{2}\Gamma(2)\zeta(4)\frac{1}{\mu^3}\frac{1}{L^3} + \frac{\pi^{-3}}{4}\Gamma(2)\zeta(4)\frac{1}{L^3} - \frac{\pi^{-\frac{9}{2}}}{4}\Gamma(\frac{5}{2})\zeta(5)\mu\frac{1}{L^3}$$
$$-\frac{1}{\mu}\frac{1}{L^3}\sum_{n_1,n_2=1}^{\infty}(\frac{n_2}{n_1})^2 K_2(2\mu n_1 n_2)$$
$$+2^{\frac{3}{2}}\pi^2(\frac{\xi}{\mu})^{\frac{3}{2}}\frac{1}{L^3}\sum_{n_1,n_2=1}^{\infty}(\frac{n_2}{n_1})^{\frac{3}{2}}K_{\frac{3}{2}}(\frac{\pi n_1 n_2}{\xi\mu})$$
$$+(2\pi^2)^{\frac{3}{2}}\xi^{\frac{1}{2}}\mu^{-\frac{5}{2}}\frac{1}{L^3}\sum_{n_1,n_2=1}^{\infty}n_1^{-\frac{1}{2}}n_2^{\frac{5}{2}}[K_{\frac{1}{2}}(\frac{\pi n_1 n_2}{\xi\mu}) + K_{\frac{5}{2}}(\frac{\pi n_1 n_2}{\xi\mu})]$$
$$+\frac{4\pi}{L^3}\sum_{k=0}^{\infty}\frac{8^{-k}(k+1)}{k!}\xi^{k+2}\prod_{j=1}^{k}[9-(2j-1)^2]\sum_{n_1,n_2,n_3=1}^{\infty}n_1^{-k-2}(\frac{\pi^2}{\mu^2}n_2^2+n_3^2)^{-\frac{k-1}{2}}$$
$$\times\exp[-\frac{n_1}{\xi}(\frac{\pi^2}{\mu^2}n_2^2+n_3^2)^{\frac{1}{2}}]$$
$$+\frac{4\pi}{L^3}\sum_{k=0}^{\infty}\frac{8^{-k}}{k!}\xi^{k+1}\prod_{j=1}^{k}[9-(2j-1)^2]\sum_{n_1,n_2,n_3=1}^{\infty}n_1^{-k-1}(\frac{\pi^2}{\mu^2}n_2^2+n_3^2)^{-\frac{k-2}{2}}$$
$$\times\exp[-\frac{n_1}{\xi}(\frac{\pi^2}{\mu^2}n_2^2+n_3^2)^{\frac{1}{2}}] \tag{10}$$

The terms with series converge very quickly [14, 27] and only the first several summands need to be taken into account for numerical calculation to further discussions.

3. The Casimri effect for parallel plates at finite temperature in the spacetime with on extra dimension

At finite temperature in the cosmological background with one UXD, we consider the sign of Casimir energy for parallel plates and the nature of Casimir force between them to present the Casimir effect clearly. The Casimir force is denoted as $F_C = -\frac{1}{L}\frac{\partial\varepsilon_C}{\partial\mu}$. We analyze the Casimir energy (10) in the limits. If the temperature is high enough $\xi \gg 1$, then

$$\varepsilon_C(\xi\gg 1) = 2^{\frac{3}{2}}\pi^2(\frac{\xi}{\mu})^{\frac{3}{2}}\frac{1}{L^3}\sum_{n_1,n_2=1}^{\infty}(\frac{n_2}{n_1})^{\frac{3}{2}}K_{\frac{3}{2}}(\frac{\pi n_1 n_2}{\xi\mu})$$
$$+(2\pi^2)^{\frac{3}{2}}\xi^{\frac{1}{2}}\mu^{-\frac{5}{2}}\frac{1}{L^3}\sum_{n_1,n_2=1}^{\infty}n_1^{-\frac{1}{2}}n_2^{\frac{5}{2}}[K_{\frac{1}{2}}(\frac{\pi n_1 n_2}{\xi\mu}) + K_{\frac{5}{2}}(\frac{\pi n_1 n_2}{\xi\mu})]$$
$$+\frac{4\pi}{L^3}\sum_{k=0}^{\infty}\frac{8^{-k}(k+1)}{k!}\xi^{k+2}\prod_{j=1}^{k}[9-(2j-1)^2]\sum_{n_1,n_2,n_3=1}^{\infty}n_1^{-k-2}(\frac{\pi^2}{\mu^2}n_2^2+n_3^2)^{-\frac{k-1}{2}}$$
$$\times\exp[-\frac{n_1}{\xi}(\frac{\pi^2}{\mu^2}n_2^2+n_3^2)^{\frac{1}{2}}]$$
$$+\frac{4\pi}{L^3}\sum_{k=0}^{\infty}\frac{8^{-k}}{k!}\xi^{k+1}\prod_{j=1}^{k}[9-(2j-1)^2]\sum_{n_1,n_2,n_3=1}^{\infty}n_1^{-k-1}(\frac{\pi^2}{\mu^2}n_2^2+n_3^2)^{-\frac{k-2}{2}}$$
$$\times\exp[-\frac{n_1}{\xi}(\frac{\pi^2}{\mu^2}n_2^2+n_3^2)^{\frac{1}{2}}]$$

$$> 0 \tag{11}$$

For a definite temperature, when the plates distance is much larger or less than the radius of UXD, the expression for Casimir energy becomes,

$$\varepsilon_C(\mu \gg 1) = -\frac{\Gamma(\frac{5}{2})\zeta(5)}{4\pi^{\frac{9}{2}}}\mu\frac{1}{L^3} \tag{12}$$

and

$$\varepsilon_C(\mu \ll 1) = -\frac{\Gamma(2)\zeta(4)}{2}\frac{1}{\mu^3}\frac{1}{L^3} \tag{13}$$

respectively. We find that the Casimir energy is an increasing function of the variable ξ. For a value of temperature there exists a maximum of Casimir energy depending on the ratio μ. The Casimir energy is calculated from equation (10). For some values of ξ, the numerical calculations lead to the data presented in Figure 1. According to (10) the special scaled temperature $\xi_0 = 0.0947$ is obtained.

If the temperature is chosen as $\xi < \xi_0$, the Casimir energy keeps negative. At $\mu = \mu_f$, the energy is equal to the maximum. Having solved equation (10), we obtain the relation between the special ratio μ_f and temperature, with μ_f growing as scaled temperature ξ as shown in Figure 2. In the spacetime with one UXD, the two parallel plates will attract each other if $\frac{R}{L} < \mu_f$, and if $\frac{R}{L} > \mu_f$, the plates will remain repulsive.

If the temperature is sufficiently high like $\xi > \xi_0$, the Casimir energy becomes positive within the range of value of μ, the ratio of plates distance to UXD size. From expression (10), we find the relations between the special ratios μ_1, μ_2 and temperature respectively shown in Figure 3. μ_1 and μ_2 are increasing and decreasing functions of scaled temperature respectively. If $\mu < \mu_1$ or $\mu > \mu_2$, the Casimir energy will be negative. The nature of the force between plates is attractive for $\mu < \mu_1$ and repulsive for $\mu > \mu_2$. If $\mu_1 < \mu < \mu_2$, the energy will keep positive.

We can apply our results above to the approaches of string theory with different size of UXDs. The environment temperature is chosen as $T = 300K$. The size of extra dimension is set L $10nm$, the suggestion of [24, 25], then $\xi \approx 1.3 \times 10^{-3} < \xi_0$, and $\mu_f \approx 5.73$. The sign of Casimir energy is negative. If the plates distance $R > 50nm$, they repulse each other or the opposite if $R < 50nm$. We also study the model with $L \sim 10^{-2}cm$ similar to [5, 6]. Certainly $\xi \approx 13 > \xi_0$, and if $\frac{R}{L} > 3.53 \times 10^9$, the nature of force between plates is repulsive while the Casimir energy keeps negative. The result $R > 3.53 \times 10^5 m$ is interesting.

4. Conclusion

In this letter we have explored the Casimir effect for the system consisting of two parallel plates at finite temperature in the Universe with extra dimensions. We derive the expression for the Casimir energy with thermal corrections and one UXD. Having studied the Casimir energy and Casimir force, we show that the Casimir energy will remain negative if the temperature is less than

a special value. We also show that owing to the sufficiently high temperature, the sign of Casimir energy will be positive when the ratio of plates distance to UXD size chosen within the range. The nature of Casimri force depends on the system structure and environment. The distance between the parallel plates is larger or less enough than the radius of UXD, their ratio more than or less than a special value, then the Casimir energy keeps negative and the plates repulse or attract each other. The two special ratios of plates distance to the radius of extra space are increasing and decreasing functions of the temperature respectively, which shows that the Casimir effect is manifest if the temperature is high but within our reach. According to our discussions on size of UXD from models of string theory, the results that the plates repulse each other under $R > 3.53 \times 10^5 m$ for $L \sim 10^{-2} cm$ at temperature $300K$ seem to be interesting.

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Figure 1: The solid, dadot, dashed curves of the Casimir energy density as functions of ratio of plates distance to UXD size for $\xi = 0.094, 0.0945, 0.096$ respectively.

Figure 2: The curve of the special ratio of plates distance to UXD size μ_f as a function of scaled temperature when $\xi < \xi_0$.

Figure 3: The curves of the special ratios of plates distance to UXD size for μ_1 and μ_2 as functions of scaled temperature when $\xi > \xi_0$.